Singularities of infinite translation surfaces

1st part: with Anja Randecker, Lucien Clavier
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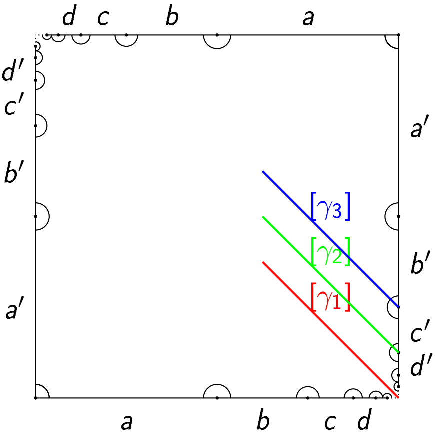
November 14, 2015

Definitions

Translation surface: 2-manifold with an atlas whose transition is translation.
Example: flat torus, flat polygon with gluing
Singularities: points added in the metric completion
We only consider the case of isolated singularities.
Types of isolated singularities: *cone-angle singularities*, *infinite singularities*, *wild singularities*

Example: Chamanara surface

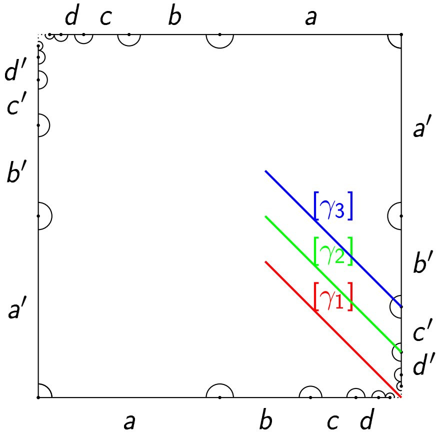


More definitions

Definition(Joshua Bowman and Ferrán Valdez): A *linear approach* at singularity σ is the equivalent class of geodesic rays $\gamma(t)$ approaching σ as $t \to 0$ up to $\sim$, where $\gamma \sim \gamma'$ iff they are identical for small t.
Bowman and Ferrán showed that the "natural" topology on the set of linear approaches $\mathcal{L}$ is generated by $B(x, r)^t = \{[\gamma] : d(\gamma(t), x) < r\}$, and is Hausdorff.

More definitions

Definition(Joshua Bowman and Ferrán Valdez): *Rotational component* is the equivalence class on $\mathcal{L}$ under the following relation R: $\gamma R \gamma'$ iff γ and γ' both lie on a embedded sector centered at σ.
We denote the set of rotational components, with quotient topology, $\widetilde{\mathcal{L}}$.



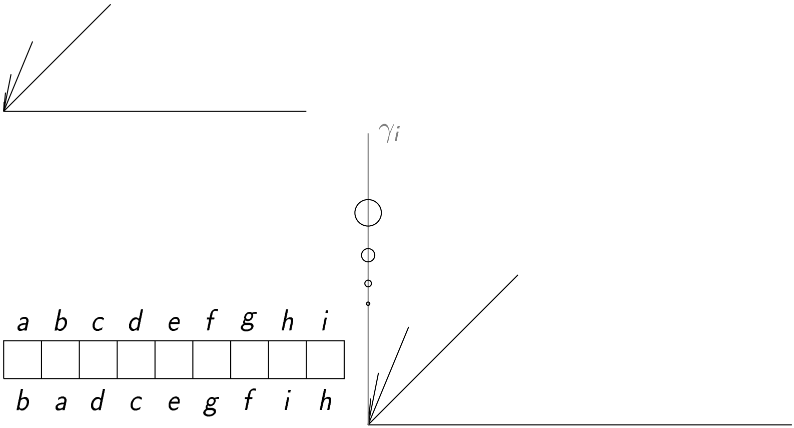
1 wild singularity, 2 rotational components of infinite length, countably infinitely many rotational components of finite length.

Properties and examples

Theorem (Clavier, Randecker, W.)

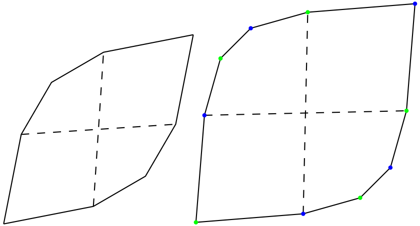
1. Rotational components isometric to $\mathbb{R}$ are open points in $\widetilde{\mathcal{L}}$; open points in $\widetilde{\mathcal{L}}$ has infinite length.
2. Points in $\widetilde{\mathcal{L}}$ with length $\geq \pi$ are dense; $\widetilde{\mathcal{L}}$ is separable.
3. $\widetilde{\mathcal{L}}$ can be homeomorphic to any topological spaces of finite points.
4. $\widetilde{\mathcal{L}}$ can be homeomorphic to $\mathbb{S}^{\nVdash}$.
5. There are uncountable many translation surfaces with one singularity, same $\widetilde{\mathcal{L}}$ and different $\mathcal{L}$.

Proof of 3.



Approximation of affine maps

1. Blow up the invariant rotational components.
2. Double.
3. Glue the ends of the mapping torus, perturb.



$$\lambda^n - 2\lambda^{n-1} + 2\lambda - 1 = 0$$

(with Baik, Rafiqi)

Other results and further questions

1. Arnoux-Yoccoz surfaces (Bowman), and many other examples
2. "Generalized" Teichmüller polynomial
3. "Limit surface"?
4. Condition on $\mathcal{L}$ and $\widetilde{\mathcal{L}}$?
5. Strata?