Constructing pseudo-Anosov homeomorphisms

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Question

The question we're trying to address is: Given a bi-Perron number λ, can we construct a compact surface S_g, and a pseudo-Anosov homeomorphism $\psi : S_g \to S_g$, whose dilatation factor is λ?

Strategy

λ-expander: a piecewise-linear continuous map from I to itself, such that the slope of each piece is either λ or $-\lambda$.
Post-critically finite: the critical points all have finite orbit.
Thurston proved the following Theorem in his paper "Entropy in Dimension 1":

Theorem (Thurston)

Given any weak Perron number λ, there is a post-critically finite λ-expander from the unit interval to itself.

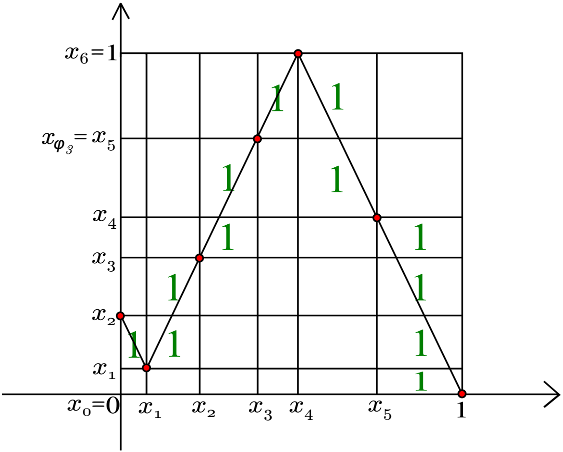
Strategy

Furthermore, he "thickened" the tent map with slope as the golden ratio, and the tent map with slope as the leading root of $\lambda^4 - \lambda^3 - \lambda^2 - \lambda + 1 = 0$ into a two dimensional piecewise-linear map, and glue their ω-limit set into closed surfaces. Carvalho-Hall also gave a similar construction for PCF tent maps.

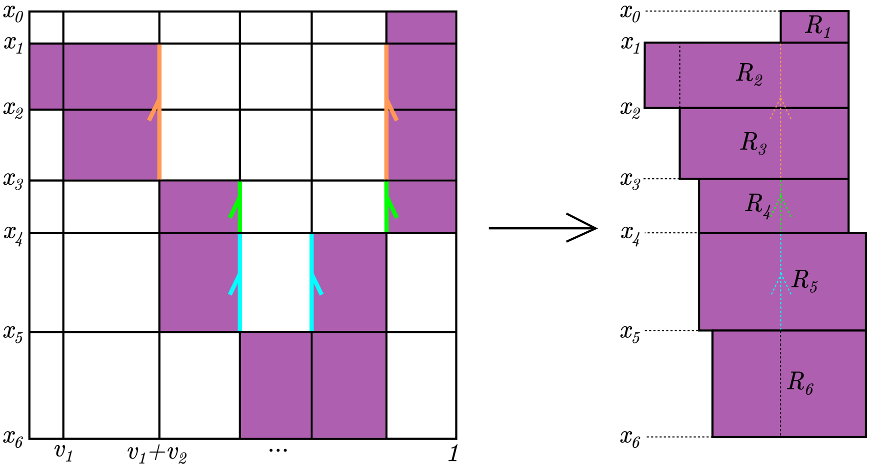
Strategy

Our strategy is to add further conditions on a PCF λ-expander that are sufficient to guarantee that its thickening is a pseudo-Anosov dilatation with λ as the eigenvalue. We will first describe a general construction, possibly yielding a surface of infinite type, and then prove that our conditions are sufficient to ensure the surface constructed is of finite type.

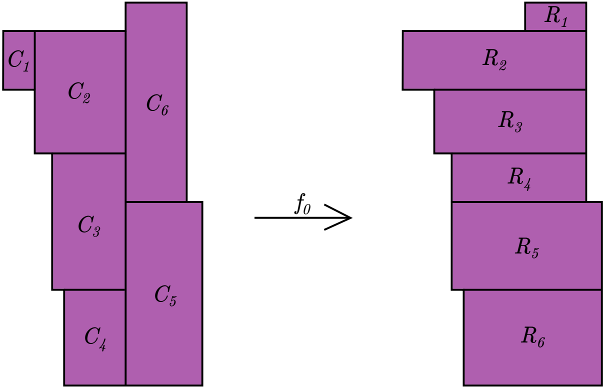
General construction



General construction



General construction



Then we glue the boundary so that f_0 and its inverse are both continuous.

The conditions

A λ-expander h can be thickened into a pseudo-Anosov map on closed surface when it:

1. Permutes the post-critical set.
2. Has an aperiodic incidence matrix.
3. Satisfies the "one sided condition": for any x in the post-critical set of h, $x = \sup h^{-1}(h(x))$ or $x = \inf h^{-1}(h(x))$.

The conditions

4. Satisfies the "alignment condition": there is a number $\epsilon \in \{-1, 1\}$, and a function α from the post-critical set X to $\{-1, 1\}$, such that:

(a) If $h^{-1}(h(x))$ has more than one point, $\alpha(x) = -1$ if $x = \inf h^{-1}(h(x))$, $\alpha(x) = 1$ if $x = \sup h^{-1}(h(x))$.

(b) For critical $x \in X$, $\alpha(i) = \begin{cases} -\epsilon & \text{if } x \text{ is a local max.} \\ +\epsilon & \text{if } x \text{ is a local min.} \end{cases}$

(c) For noncritical $x \in X$, $\alpha(i) = \begin{cases} +\epsilon\, \alpha(h(x)) & \text{if } h'(x) > 0. \\ -\epsilon\, \alpha(h(x)) & \text{if } h'(x) < 0. \end{cases}$

All 4 conditions are very easy to check computationally.