Singularity of infinite translation surface

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Infinite translation surface

Translation surface: surface with a chart whose coordinate transforms are all translations.
Infinite translation surface: Translation surface whose metric completion is not a closed surface w/ an abelian differential.
Application:

1. Infinite IET
2. Piecewise linear maps on union of rectangles

Linear approaches and rotational components

Joshua Bowman and Ferrán Valdez introduced the following concepts in their 2013 paper "Wild singularities of flat surfaces" to characterize the topology of isolated singularities of infinite translation surfaces.

Linear approaches

Definition (Linear approaches)

Let (X, ω) be a translation surface and $\epsilon > 0$. On the space

$$\mathcal{L}^{\epsilon}(X) = \{\gamma : (0, \epsilon) \to X : \gamma \text{ is a geodesic curve}\}$$

we define the following equivalence relation: $\gamma_1 \in \mathcal{L}^{\epsilon}(X)$ and $\gamma_2 \in \mathcal{L}^{\epsilon'}(X)$ are called equivalent if $\gamma_1(t) = \gamma_2(t)$ for all $t \in (0, \min\{\epsilon, \epsilon'\})$.
The space

$$\mathcal{L}(X) = \bigsqcup_{\epsilon > 0} \mathcal{L}^{\epsilon}(X) / \sim$$

is called *space of linear approaches of* X and the equivalence class $[\gamma]$ of $\gamma \in \mathcal{L}^{\epsilon}(X)$ is called *linear approach*.

We give $\mathcal{L}^{\epsilon}$ the uniform topology and $\mathcal{L}$ the direct limit topology.

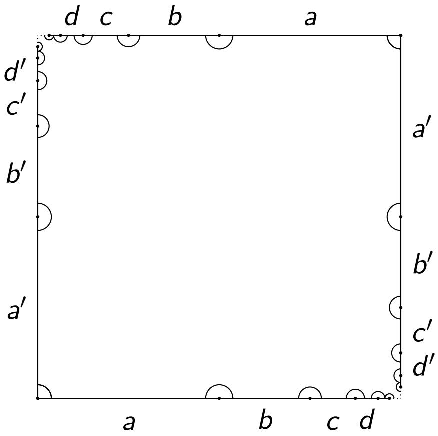
Rotational components

An angular sector is a triple (I, c, i_c) such that I is a non-empty interval, $c \in \mathbb{R}$ and i_c is an isometry from $\{x + \mathrm{i}y | x < c, y \in I\} \subseteq \mathbb{C}$ with metric defined by $e^z dz$ to X. Define an equivalence relation $\sim$ as follows: two linear approaches $[\gamma_1]$ and $[\gamma_2]$ are related by $\sim$ if there is an angular sector (I, c, i_c) such that i_c^{-1} of the images of γ_1 and γ_2 are two rays parallel to the real axis.

Definition (Rotational components)

An equivalence class under $\sim$ is called a *rotational component*. We define the *space of rotational components* as $\widetilde{\mathcal{L}}(x) = \mathcal{L}(x)/\sim$. We give $\widetilde{\mathcal{L}}$ the quotient topology.

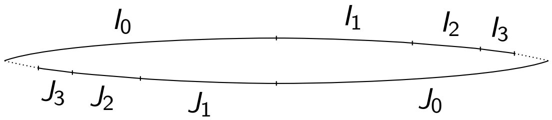
Example 1



Chamanara surface: segments with the same letters are glued.

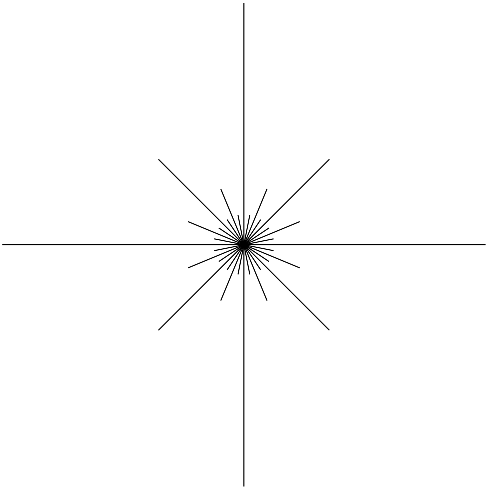
Reza Chamanara, "Affine automorphism groups of surfaces of infinite type", 2014

Example 2



Geometric series decoration

Example 3

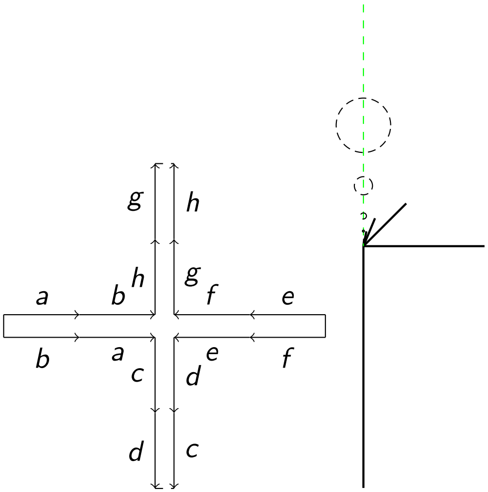


Star decoration

Example 4

Shrinking star decoration

Example 4, continued



Example 5